History

Main article: History of trigonometric functions

While the early study of trigonometry can be traced to antiquity, the trigonometric functions as they are in use today were developed in the medieval period. The chord function was discovered by Hipparchus of Nicaea (180–125 BC) and Ptolemy of Roman Egypt (90–165 AD).

The functions sine and cosine can be traced to the *jyā* and *koti-jyā* functions used in Gupta period Indian astronomy (*Aryabhatiya*, *Surya Siddhanta*), via translation from Sanskrit to Arabic and then from Arabic to Latin.[23]

All six trigonometric functions in current use were known in Islamic mathematics by the 9th century, as was the law of sines, used in solving triangles.[24] al-Khwārizmī produced tables of sines, cosines and tangents. They were studied by authors including Omar Khayyám, Bhāskara II, Nasir al-Din al-Tusi, Jamshīd al-Kāshī (14th century), Ulugh Beg (14th century), Regiomontanus (1464), Rheticus, and Rheticus' student Valentinus Otho.

Madhava of Sangamagrama (c. 1400) made early strides in the analysis of trigonometric functions in terms of infinite series.[25]

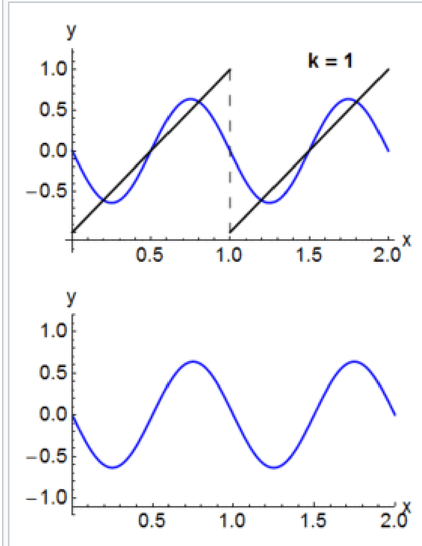
The terms *tangent* and *secant* were first introduced in 1583 by the Danish mathematician Thomas Fincke in his book *Geometria rotundi*.[26]

The first published use of the abbreviations *sin*, *cos*, and *tan* is by the 16th century French mathematician Albert Girard.

In a paper published in 1682, Leibniz proved that $\sin x$ is not an algebraic function of x.[27]

Leonhard Euler's *Introductio in analysin infinitorum* (1748) was mostly responsible for establishing the analytic treatment of trigonometric functions in Europe, also defining them as infinite series and presenting "Euler's formula", as well as the near-modern abbreviations *sin., cos., tang., cot., sec.,* and *cosec.*[28]

A few functions were common historically, but are now seldom used, such as the chord ($\operatorname{crd}(\theta) = 2\sin(\frac{\theta}{2})$), the versine ($\operatorname{versin}(\theta) = 1 - \cos(\theta) = 2\sin^2(\frac{\theta}{2})$) (which appeared in the earliest tables[28]), the haversine ($\operatorname{haversin}(\theta) = \frac{1}{2}\operatorname{versin}(\theta) = \sin^2(\frac{\theta}{2})$), the exsecant ($\operatorname{exsec}(\theta) = \sec(\theta) - 1$) and the excosecant ($\operatorname{excsc}(\theta) = \operatorname{exsec}(\frac{\pi}{2} - \theta) = \csc(\theta) - 1$). Many more relations between these functions are listed in the article about trigonometric identities.



Sinusoidal basis functions (bottom) can form a sawtooth wave (top) when added. All the basis functions have nodes at the nodes of the sawtooth, and all but the fundamental ($k = 1$) have additional nodes. The oscillation seen about the sawtooth when k is large is called the Gibbs phenomenon

Etymology

The word *sine* derives[29] from Latin *sinus*, meaning "bend; bay", and more specifically "the hanging fold of the upper part of a toga", "the bosom of a garment", which was chosen as the translation of what was interpreted as the Arabic word *jaib*, meaning "pocket" or "fold" in the twelfth-century translations of works by Al-Battani and al-Khwārizmī into Medieval Latin.[30] The choice was based on a misreading of the Arabic written form *j-y-b* (جيب), which itself originated as a transliteration from Sanskrit *jīvā*, which along with its synonym *jyā* (the standard Sanskrit term for the sine) translates to "bowstring", being in turn adopted from Ancient Greek χορδή "string".[31]

The word *tangent* comes from Latin *tangens* meaning "touching", since the line *touches* the circle of unit radius, whereas *secant* stems from Latin *secans* — "cutting" — since the line *cuts* the circle.[32]

The prefix "co-" (in "cosine", "cotangent", "cosecant") is found in Edmund Gunter's *Canon triangulorum* (1620), which defines the *cosinus* as an abbreviation for the *sinus complementi* (sine of the complementary angle) and proceeds to define the *cotangens* similarly.[33]

See also

- All Students Take Calculus — a mnemonic for recalling the signs of trigonometric functions in a particular quadrant of a Cartesian plane
- Aryabhata's sine table

- Aryabhata's sine table
- Bhaskara I's sine approximation formula
- Generalized trigonometry
- Generating trigonometric tables
- Hyperbolic function
- List of periodic functions
- List of trigonometric identities
- Madhava series
- Madhava's sine table
- Polar sine — a generalization to vertex angles
- Proofs of trigonometric identities
- Versine — for several less used trigonometric functions

Notes

1. ↑ Oxford English Dictionary, sine, *n.*[2]
2. ↑ Oxford English Dictionary, cosine, *n.*
3. ↑ Oxford English Dictionary, tangent, *adj. and n.*
4. ↑ Oxford English Dictionary, secant, *adj.* and *n.*
5. ↑ Heng, Cheng and Talbert, "Additional Mathematics", page 228
6. ↑ Ron Larson, Ron (2013). *Trigonometry* (9th ed.). Cengage Learning. p. 153. ISBN 978-1-285-60718-4. Extract of page 153
7. ↑ See Ahlfors, pages 43–44.
8. ↑ Abramowitz; Weisstein.
9. ↑ Stanley, Enumerative Combinatorics, Vol I., page 149
10. ↑ Aigner, Martin; Ziegler, Günter M. (2000). *Proofs from THE BOOK* (Second ed.). Springer-Verlag. p. 149. ISBN 978-3-642-00855-9.
11. ↑ Remmert, Reinhold (1991). *Theory of complex functions*. Springer. p. 327. ISBN 0-387-97195-5. Extract of page 327
12. ↑ For a demonstration, see Euler's formula#Using power series
13. ↑ Needham, Tristan. *Visual Complex Analysis*. ISBN 0-19-853446-9.
14. ↑ Kannappan, Palaniappan (2009). *Functional Equations and Inequalities with Applications*. Springer. ISBN 978-0387894911.
15. ↑ Kantabutra.
16. ↑ However, doing that while maintaining precision is nontrivial, and methods like Gal's accurate tables, Cody and Waite reduction, and Payne and Hanek reduction algorithms can be used.
17. ↑ Brent, Richard P. (April 1976). "Fast Multiple-Precision Evaluation of Elementary Functions". *J. ACM*. **23** (2): 242–251. ISSN 0004-5411. doi:10.1145/321941.321944.
18. ↑ Abramowitz, Milton and Irene A. Stegun, p.74
19. ↑ The Universal Encyclopaedia of Mathematics, Pan Reference Books, 1976, page 529. English version George Allen and Unwin, 1964. Translated from the German version Meyers Rechenduden, 1960.
20. ↑ The Universal Encyclopaedia of Mathematics, Pan Reference Books, 1976, page 530. English version George Allen and Unwin, 1964. Translated from the German version Meyers Rechenduden, 1960.
21. ↑ Stanley J Farlow (1993). *Partial differential equations for scientists and engineers* (Reprint of Wiley 1982 ed.). Courier Dover Publications. p. 82. ISBN 0-486-67620-X.
22. ↑ See for example, Gerald B Folland (2009). "Convergence and completeness". *Fourier Analysis and its Applications* (Reprint of Wadsworth & Brooks/Cole 1992 ed.). American Mathematical Society. pp. 77ff. ISBN 0-8218-4790-2.
23. ↑ Boyer, Carl B. (1991). A History of Mathematics (Second ed.). John Wiley & Sons, Inc.. ISBN 0-471-54397-7, p. 210.
24. ↑ Owen Gingerich (1986). "Islamic Astronomy". **254**. Scientific American: 74. Archived from the original on 2013-10-19. Retrieved 2010-07-13.
25. ↑ J J O'Connor and E F Robertson. "Madhava of Sangamagrama". School of Mathematics and Statistics University of St Andrews, Scotland. Retrieved 2007-09-08.
26. ↑ "Fincke biography". Retrieved 15 March 2017.
27. ↑ Nicolás Bourbaki (1994). *Elements of the History of Mathematics*. Springer.
28. ↑ [28.0] [28.1] See Boyer (1991).
29. ↑ The anglicized form is first recorded in 1593 in Thomas Fale's *Horologiographia, the Art of Dialling*.

30. ↑ various sources credit the first use of *sinus* to either
    - Plato Tiburtinus's 1116 translation of the *Astronomy* of Al-Battani
    - Gerard of Cremona's translation of the *Algebra* of al-Khwārizmī
    - Robert of Chester's 1145 translation of the tables of al-Khwārizmī

    See Merlet, *A Note on the History of the Trigonometric Functions* in Ceccarelli (ed.), *International Symposium on History of Machines and Mechanisms*, Springer, 2004
    See Maor (1998), chapter 3, for an earlier etymology crediting Gerard.
    See Katx, Victor (July 2008). *A history of mathematics* (3rd ed.). Boston: Pearson. p. 210 (sidebar). ISBN 978-0321387004.
31. ↑ See Plofker, *Mathematics in India*, Princeton University Press, 2009, p. 257
    See "Clark University".
    See Maor (1998), chapter 3, regarding the etymology.
32. ↑ Oxford English Dictionary
33. ↑ OED. The text of the *Canon triangulorum* as reconstructed may be found here

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External links

- Hazewinkel, Michiel, ed. (2001), "Trigonometric functions", *Encyclopedia of Mathematics*, Springer, ISBN 978-1-55608-010-4
- Visionlearning Module on Wave Mathematics
- GonioLab Visualization of the unit circle, trigonometric and hyperbolic functions

Wikibooks has a book on the topic of: ***Trigonometry***

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