Sine, cosine, and tangent

The sine of an angle is the ratio of the length of the opposite side to the length of the hypotenuse. The word comes from the Latin sinus for gulf or bay, since, given a unit circle, it is the side of the triangle on which the angle opens. In our case:

\[ \sin A = \frac{\text{opposite}}{\text{hypotenuse}} \]

The cosine of an angle is the ratio of the length of the adjacent side to the length of the hypotenuse, so called because it is the sine of the complementary or co-angle, the other non-right angle. Because the angle sum of a triangle is \( \pi \) radians, the co-angle \( B \) is equal to \( \frac{\pi}{2} - A \); so \( \cos A = \sin B = \sin(\frac{\pi}{2} - A) \). In our case:

\[ \cos A = \frac{\text{adjacent}}{\text{hypotenuse}} \]

The tangent of an angle is the ratio of the length of the opposite side to the length of the adjacent side: so called because it can be represented as a line segment tangent to the circle, that is the line that touches the circle, from Latin \( \text{linea tangens} \) or touching line (cf. tangere, to touch). In our case:

\[ \tan A = \frac{\text{opposite}}{\text{adjacent}} \]

Tangent may also be represented in terms of sine and cosine, that is:

\[ \tan A = \frac{\sin A}{\cos A} = \frac{\text{opposite}}{\text{adjacent}} \]

These ratios do not depend on the size of the particular right triangle chosen, as long as the focus angle is equal, since all such triangles are similar.

The acronyms "SOH-CAH-TOA" ("soak-a-toe", "sock-a-toa", "so-kah-toa") and "OHSAHOAT" are commonly used trigonometric mnemonics for these ratios.

Cosecant, secant, and cotangent

The remaining three functions are best defined using the above three functions, and can be considered their reciprocals.

The cosecant \( \csc(A) \) or cosec(\( A \)), is the reciprocal of \( \sin(A) \); i.e. the ratio of the length of the hypotenuse to the length of the opposite side; so called because it is the secant of the complementary or co-angle:

\[ \csc A = \frac{1}{\sin A} = \frac{\text{hypotenuse}}{\text{opposite}} = \frac{h}{a}. \]

The secant \( \sec(A) \) is the reciprocal of \( \cos(A) \); i.e. the ratio of the length of the hypotenuse to the length of the adjacent side: